UOFSC, DEPARTMENT OF PHYSICS & ASTRONOMY. Graduate student problem competition SPT 7–30, 2022

All graduate students are eligible to participate. To submit your solution, e-mail it to bazaliy@mailbox.sc.edu

Charging a conductor

Two separate metal objects reside far away from all other charges and conductors. A small metal sphere is initially also far away from the two objects. A charge Q is placed on it, after which it is brought from infinity into contact with object #2. Upon such contact, a certain fraction of charge flows from the sphere to #2. The charge on object #1 does not change since it is never touched by the sphere. Then the sphere is removed away to infinity, charged up to the same Q, and brought back in contact with #2. The cycle can be repeated any number of times. Each time the contact of the sphere and #2 happens with all three objects being in the same position relative to each other.

It is known that if initially both objects are uncharged, the first contact transfers charge αQ to #2. It is further known that if initially #1 has charge Q (same charge as the sphere receives on each cycle) and #2 is uncharged, then upon the first contact charge βQ is transferred from the sphere to #2.

Suppose now #1 has the charge -Q. The sphere is repeatedly shuttled between infinity and #2, until the charge on #2 stops changing. Find the limiting value of charge on #2.