## THE SIMPLE PENDULUM

## Objective

To understand the relationship between the period and the length of a simple pendulum.

## Equipment

Pendulum apparatus, balance, thin string, 2-pendulum bobs, 2 lab posts and clamps, Smart Timer, photogate, protractor, meter stick or metric tape measure.

## Introduction

A simple pendulum consists of a point mass suspended at the end of a cord of zero mass. A close approximation to this is a small metal mass on a long, light string or thread. In this experiment you will time how long it takes to swing back and forth. The time for one complete swing is the period. The only variables you have are: (1) the mass, (2) the length of the string, and (3) the amplitude (that is, the size of the angle that the pendulum swings through). In this experiment, you will be varying the length of the string for two different masses.


Figure 1: A simple pendulum. The symbols are: $m$ is the mass, $L$ is the length, $\theta$ is the angle of amplitude, and $x$ is the horizontal displacement.

The angle of amplitude, $\theta$, may be measured with a protractor or calculated from the relationship:

$$
\begin{equation*}
\theta \approx \frac{x}{L} \tag{1}
\end{equation*}
$$

This computation gives the angle in units of radians. The conversion to/from degrees is 1 radian $=57.3^{\circ}$ and $1^{\circ}=0.0175 \mathrm{rad}$.

Detailed analysis shows that for the simple pendulum. the period, $T$, when $\theta$ is kept small, is given by:

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{L}{g}} \tag{2}
\end{equation*}
$$

where $g$ is the gravitational constant. If we square both sides of equation and rearrange, we can get

$$
\begin{equation*}
4 \pi^{2} L=g T^{2} \tag{3}
\end{equation*}
$$

We will use this form of the period equation when we plot our data.

## Activity 1.

1) In this experiment, we will be using two different mass bobs and investigating the effect of mass on the period. Using a balance, measure the mass of each bob. Record these measurements on your data sheet. The measurements should be in grams (g), convert your mass values to kilograms ( kg ) and record on your data sheet.

## Activity 2.

2) Insert the lab post into the table so it is vertical, and clamp the second post to this so that is is horizontal. Attach the pendulum apparatus to the horizontal lab post and suspend on of the pendulum bobs (either wood or brass). There is a clamp with a thumbscrew you can use to suspend the bob. This will allow the pendulum to oscillate about a single point. Align the pendulum bob at each of your lengths of string so that the bob passes through the photo gate to permit a period measurement.
3) Set the apparatus so that the length of the pendulum is between 5 and 80 cm . Accurately measure and record the length you use on your data sheet. For these measurements, you should keep the amplitude of oscillation small, less than about $10^{\circ}$. This can be estimated by eye, but better still, a protractor for consistency.
4) On the Smart Timer select measurement for Time, set the mode to pendulum. Start the bob swinging, in an arc of about 10 degrees. Record the starting angle on the data sheet. At the extreme of the swing press start. The meter will display the time for one complete cycle. Note, one cycle is defined as the motion that returns the bob to its original position and velocity.
5) Measure and record the time required for the bob to complete a cycle (the period) at least 5 times. It is best if you start the bob swinging, make sure it is smooth, and then start recording. Starting the swing can introduce other oscillations, which may affect your readings. Thus, it is best to measure your data with only one release.

## Record all 5 period measurements on the data sheet.

Calculate the average period and record on the data sheet.

## Activity 3.

6) Measure the period of the pendulum using a variety of different pendulum lengths ranging from 5 cm to 80 cm . Choose at least 7 different lengths in this range. Your choices should span the entire range.
7) Repeat the entire experiment for the second pendulum bob.
8) Calculate the average period for each run of different string length. Calculate the square of this average time. To prepare the data we will need to make a plot, multiply the pendulum length by $4 \pi^{2}$. Record all these values to tables 5 and 6 on your data sheet.

## Activity 4.

9) Using a spreadsheet, preferably Microsoft Excel, or other graphing software, prepare a graph of the length $4 \pi^{2} L$ versus $T^{2}$ for both mass bobs (on the same graph). Make sure to label the axes and give your plot a title.
10) Using the trend line function, add linear trend lines to your plot. Make sure you turn on the trend line values to see the equation of the line.
11) From equation 3 above, we can see that the slope of the trend lines will be experimental values for the gravitational acceleration, $g$. Record your experimental values for $g$ on your data table.

## Activity 5.

12) Compare your experimental values of $g$ to the standard value of $9.81 \mathrm{~m} / \mathrm{s}^{2}$ by computing the percent error. The percent error is given by the difference between your result and the standard value divided by the standard value and converted to a percent.

$$
\begin{equation*}
\text { percent error }=\left|\frac{\text { experimental value }- \text { standard value }}{\text { standard value }}\right| * 100 \tag{4}
\end{equation*}
$$

## Summary.

Answer the following questions and include with your data sheet:
Looking at your plot, did the mass of the bob have any effect on the period?
Did the bob material (brass or wood) make any difference?
Looking at your plot and the percent error values, is equation 2 for the period of a simple pendulum validated?

Instead of using the timer to measure the period of many individual oscillations, how would you find the period with good accuracy using a stopwatch??

## THE SIMPLE PENDULUM

## DATA SHEET

## Activity 1.

Table 1: Record the mass of the bobs.

|  | Mass (g) | Mass (kg) |
| :--- | :--- | :--- |
| Brass Bob |  |  |
| Wooden Bob |  |  |

## Activity 2.

Pendulum length (m): $\qquad$
Starting angle (degrees): $\qquad$

Table 2: Data table for Activity 2.

| Length $(\mathrm{m})$ | Trial 1 (s) | Trial 2 (s) | Trial 3 (s) | Trial 4 (s) | Trial 5 (s) |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

Average period (s): $\qquad$

## Activity 3.

Table 3: Generate data table for brass mass bob.

| Length (m) | Trial 1 (s) | Trial 2 (s) | Trial 3 (s) | Trial 4 (s) | Trial 5 (s) |
| :--- | :--- | :--- | :--- | :--- | :--- |
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Mass of bob 1: $\qquad$

Table 4: Generate data table for wooden mass bob.

| Length (m) | Trial 1 (s) | Trial 2 (s) | Trial 3 (s) | Trial 4 (s) | Trial 5 (s) |
| :--- | :--- | :--- | :--- | :--- | :--- |
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Mass of bob 2: $\qquad$

Table 5: Table for plot data, brass mass bob.

| Length (m) | Average T (s) | $\left(\right.$ Average T) ${ }^{2}\left(\mathrm{~s}^{2}\right)$ | $\mathrm{L}^{*} 4 \pi^{2}(\mathrm{~m})$ |
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Table 6: Table for plot data, wooden mass bob.

| Length (m) | Average T (s) | $\left(\right.$ Average T) ${ }^{2}\left(\mathrm{~s}^{2}\right)$ | $\mathrm{L}^{*} 4 \pi^{2}(\mathrm{~m})$ |
| :--- | :--- | :--- | :--- |
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## Activity 4.

Include a copy of your plot with this assignment. Make sure your plot is in the proper format. For example, are your axes labeled and show the proper units?

Experimental value for $g$ (brass bob): $\qquad$
Experimental value for $g$ (wooden bob):

## Activity 5.

Compare your experimental value for $g$ to the standard value of $9.81 \mathrm{~m} / \mathrm{s}^{2}$ by computing a percent error.

Percent Error (brass bob): $\qquad$
Percent Error (wooden bob): $\qquad$

Answer the questions from the summary.

